Absolute stability of axisymmetric perturbations in strongly-magnetized collisionless axisymmetric accretion disk plasmas

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(Dated: January 10, 2012)

The physical mechanism responsible for driving accretion flows in astrophysical accretion disks is commonly thought to be related to the development of plasma instabilities and turbulence. A key question is therefore the determination of consistent equilibrium configurations for accretion-disk plasmas and investigation of their stability properties. In the case of collisionless plasmas kinetic theory provides the appropriate theoretical framework. This paper presents a kinetic description of low-frequency and long-wavelength axisymmetric electromagnetic perturbations in non-relativistic, strongly-magnetized and gravitationally-bound axisymmetric accretion-disk plasmas in the collisionless regime. The analysis, carried out within the framework of the Vlasov-Maxwell description, relies on stationary kinetic solutions of the Vlasov equation which allow for the simultaneous treatment of non-uniform fluid fields, stationary accretion flows and temperature anisotropies. It is demonstrated that, for such solutions, no axisymmetric unstable perturbations can exist occurring on characteristic time and space scales which are long compared with the Larmor gyration time and radius. Hence, these stationary configurations are actually stable against axisymmetric kinetic instabilities of this type. As a fundamental consequence, this rules out the possibility of having the axisymmetric magneto-rotational or thermal instabilities to arise in these systems.

A fundamental issue in the physics of accretion disks (ADs) concerns the stability of equilibrium or quasi-stationary configurations occurring in AD plasmas. The observed transport phenomena giving rise to the accretion flow are commonly ascribed to the existence of instabilities and the subsequent development of fluid or MHD turbulence [1–4]. In principle, these can include both MHD phenomena (such as drift instabilities driven by gradients of the fluid fields) and kinetic ones (due to velocity-space anisotropies, including, for example, trapped-particle modes, cyclotron and Alfven waves, etc.). Possible candidates for the angular momentum transport mechanism are usually identified either with the magneto-rotational instability (MRI) [5–7] or the thermal instability (TMI) [8–11], caused by unfavorable gradients of rotation/shear and temperature respectively. The validity of the above identifications needs to be checked in this case, because they usually rely on incomplete physical descriptions, which ignore the microscopic (kinetic) plasma behavior. In fact, "stand-alone" fluid and MHD approaches which are not explicitly based on kinetic theory and/or do not start from consistent kinetic equilibria, may become inadequate or inapplicable for collisionless or weakly-collisional plasmas. Apart from possible gyrokinetic and finite Larmor-radius effects (which are typically not included for MRI and TMI), this concerns consistent treatment of the kinetic constraints which must be imposed on the fluid fields (see related discussion in Refs.[12, 13]). This concerns, in particular, the correct determination of the constitutive equations for the relevant fluid fields. Because of this, the issue of stability of these systems is in need of further study.

In this regard, some relevant background materials are provided by Refs.[12–14], where a perturbative kinetic theory for collisionless plasmas has been developed and the existence of asymptotic kinetic equilibria has been demonstrated for axisymmetric magnetized plasmas. In AD plasmas, in particular, they are characterized by the presence of *stationary azimuthal and poloidal species-dependent flows* and can support *stationary kinetic dynamo effects*, responsible for the self-generation of azimuthal and poloidal magnetic fields [15], together with *stationary accretion flows.* This provides the basis for a systematic stability analysis of such systems. We stress that these features arise as part of the kinetic equilibrium solution, and are not dependent on perturbative instabilities. Furthermore, by assumption in the theory developed here there is no background (i.e., externally-produced) radiation field. In principle, for a collisionless plasma at equilibrium, charged particles can be still subject to EM radiation produced by accelerating particles (EM radiation-reaction). However, the effect of these physical mechanisms is negligible for the dynamics of non-relativistic plasmas, and therefore they can be safely ignored in the present treatment.

In this paper we address the stability of these equilibria with respect to infinitesimal axisymmetric perturbations. We restrict attention to the treatment of non-relativistic, strongly-magnetized and gravitationally-bound (see definition below) collisionless AD plasmas around compact objects for which the theory developed in Refs.[12, 13] applies. The plasmas can be considered *quasi-neutral* and characterized by a *mean-field interaction.* Accretion disks fulfilling these requirements rely necessarily on kinetic theory in the so-called Vlasov-Maxwell statistical description, which represents the fundamental physical approach for these systems. In AD plasmas, electromagnetic (EM) fields can be present, which may either be externally produced or self-generated. At equilibrium, they are taken here to be axisymmetric and of the general form $\mathbf{B}^{(eq)} \equiv B^{(eq)}\mathbf{b} = \mathbf{B}_T^{(eq)} + \mathbf{B}_P^{(eq)}$ and $\mathbf{E}^{(eq)} \equiv -\nabla\Phi^{(eq)}(\mathbf{x})$. Here $\mathbf{B}_T^{(eq)} \equiv I(\mathbf{x})\nabla\varphi$ and $\mathbf{B}_P^{(eq)} \equiv \nabla\psi(\mathbf{x}) \times \nabla\varphi$ are the toroidal and poloidal components of the magnetic field respectively, with $I(\mathbf{x})$ and $\Phi^{(eq)}(\mathbf{x})$ being the toroidal current and the electrostatic potential. Furthermore, (R, φ, z) denote a set of cylindrical coordinates, with $\mathbf{x} = (R, z)$, while $(\psi, \varphi, \vartheta)$ is a set of local magnetic coordinates, with ψ being the so-called poloidal flux function. The validity of the previous representation for $\mathbf{B}^{(eq)}$ requires the existence of locally nested magnetic $\psi-$surfaces, represented by $\psi = const.$, while the expressions for $\psi(\mathbf{x})$, $I(\mathbf{x})$ and $\Phi^{(eq)}(\mathbf{x})$ follow from the stationary Maxwell equations. The gravitational field is treated here non-relativistically, by means of the gravitational potential $\Phi_G = \Phi_G(\mathbf{x})$. This means that the electrostatic and gravitational fields are formally replaced by the effective electric field $\mathbf{E}_s^{eff} = -\nabla\Phi_s^{eff}$, determined in terms of the effective electrostatic potential $\Phi_s^{eff} = \Phi^{(eq)}(\mathbf{x}) + \frac{M_s}{Z_s e}\Phi_G(\mathbf{x})$, with M_s and $Z_s e$ denoting the mass and charge, respectively, of the s-species particle (where s can indicate either ions or electrons). Based on astronomical observations, the magnetic field magnitudes are expected to range in the interval $B \sim 10^1 - 10^8 G$ [16–18]. This implies that the proton Larmor radius r_{Li} is in the range $10^{-6} - 10^3 cm$ (the lower values corresponding to the lower temperature and the higher magnetic field). Additional important physical parameters are related to the species number density and temperature. Astrophysical AD plasmas can have a wide range of values for the particle number density n_s, depending on the circumstances considered. Here we focus on the case of collisionless and non-relativistic AD plasmas assuming values of the number density n_s in the range $n_s \sim 10^6 - 10^{15} cm^{-3}$. For reference, the highest value of this interval corresponds to ion mass density $\rho_i \sim 10^{-9} gcm^{-3}$. The choice of this parameter interval lies well in the range of values which can be estimated for the so-called radiatively inefficient accretion flows (RIAFs, [16, 19]). For these systems, estimates for species temperatures usually lie in the ranges $T_i \sim 1 - 10^5 keV$ and $T_e \sim 1 - 10 keV$ for ions and electrons respectively. Depending on the magnitude of the EM, gravitational and fluid fields, the AD plasmas can sustain a variety of notable physical phenomena, the systematic treatment of which requires their classification in terms of suitable dimensionless parameters. These are identified with $\varepsilon_{M,s}$, ε_s and σ_s, to be referred to as *Larmor-radius*, *canonical momentum* and *total-energy parameters*. Their definitions are respectively: $\varepsilon_{M,s} \equiv \frac{r_{Ls}}{(\Delta L)^{eq}}$, $\varepsilon_s \equiv \left|\frac{M_s R v_\varphi}{\frac{Z_s e}{c}\psi}\right|$ and $\sigma_s \equiv \left|\frac{\frac{M_s}{2}v^2}{Z_s e\Phi_s^{eff}}\right|$. Here, $r_{Ls} \equiv v_{ths}/\Omega_{cs}$ denotes the Larmor radius of the species s, v_{ths} and Ω_{cs} are the species thermal velocity and the Larmor frequency respectively, $(\Delta L)^{eq}$ is the characteristic scale-length of the equilibrium fluid fields, $\mathbf{v}$ is the single-particle velocity and $v_\varphi \equiv \mathbf{v} \cdot R\nabla\varphi$. Systems satisfying the asymptotic ordering $0 \leq \sigma_s, \varepsilon_s, \varepsilon, \varepsilon_{M,s} \ll 1$ are referred to as *strongly-magnetized and gravitationally-bound* plasmas [12, 13], with the parameters σ_s, ε_s and $\varepsilon_{M,s}$ to be considered as independent while $\varepsilon \equiv \max\{\varepsilon_s, s = 1, n\}$. In the following, we shall assume that the poloidal flux is of the form $\psi \equiv \frac{1}{\varepsilon}\overline{\psi}(\mathbf{x})$, with $\overline{\psi}(\mathbf{x}) \sim O(\varepsilon^0)$, while the equilibrium electric field satisfies the constraint $\frac{\mathbf{E}^{(eq)} \cdot \mathbf{B}^{(eq)}}{\left|\mathbf{E}^{(eq)}\right|\left|\mathbf{B}^{(eq)}\right|} \sim O(\varepsilon)$. This implies that to leading-order $\Phi^{(eq)}$ is a function of ψ only, while Φ_s^{eff} remains generally a function of the type $\Phi_s^{eff} = \overline{\Phi}_s^{eff}(\overline{\psi}, \vartheta)$ (see Ref.[13]). At equilibrium, by construction, the particle toroidal canonical momentum $p_{\varphi s} \equiv \frac{Z_s e}{c}\psi_{*s} = M_s R v_\varphi + \frac{Z_s e}{c}\psi$, the total particle energy $E_s \equiv Z_s e\Phi_{*s} = \frac{M_s}{2}v^2 + Z_s e\Phi_s^{eff}$ and the magnetic moment m'_s predicted by gyrokinetic theory are either exact or adiabatic invariants. In particular, the above orderings imply the leading-order asymptotic perturbative expansions for the variables ψ_{*s} and Φ_{*s}:

$$\psi_{*s} = \psi\left[1 + O(\varepsilon_s)\right], \tag{1}$$

$$\Phi_{*s} = \Phi_s^{eff}\left[1 + O(\sigma_s)\right], \tag{2}$$

while similarly $m'_s = \frac{M_s w'^2}{2B'}\left[1 + O(\varepsilon_{M,s})\right]$. From here on, we will use the notation that primed quantities are always evaluated at the guiding-center. In particular, $\mathbf{w}' = \mathbf{v} - u'\mathbf{b}' - \mathbf{V}'_{eff}$ denotes the perpendicular particle velocity in the local frame having the effective drift velocity $\mathbf{V}'_{eff} \equiv \frac{c}{B'}\mathbf{E}_s^{eff} \times \mathbf{b}'$, while $u' \equiv \mathbf{v} \cdot \mathbf{b}'$. In the following we shall also assume that the toroidal and poloidal magnetic fields and the species accretion and azimuthal flow velocities scale as $\frac{|\mathbf{B}_T|}{|\mathbf{B}_P|} \sim O(\varepsilon)$ and $\frac{|\mathbf{V}_{accr,s}|}{|\mathbf{V}_{\varphi,s}|} \sim O(\varepsilon)$ respectively.

In validity of the previous assumptions, an explicit asymptotic solution of the Vlasov equation can be obtained for the kinetic distribution function f_{*s}^{eq} (KDF). As pointed out in Ref.[13], ignoring slow-time dependencies, this is of the generic form $f_{*s}^{eq} = f_{*s}^{eq}(X_{*s}, (\psi_{*s}, \Phi_{*s}))$. Here X_{*s} are the invariants $X_{*s} \equiv \left(E_s, \psi_{*s}, p'_{\varphi s}, m'_s\right)$, while the brackets

(ψ_{*s}, Φ_{*s}) denote implicit dependencies for which the perturbative expansions (1) and (2) are performed. Therefore, f_{*s}^{eq} is by construction an adiabatic invariant, defined on a subset of the phase-space $\Gamma = \Omega \times U$, with $\Omega \subset \mathbb{R}^3$ and $U \equiv \mathbb{R}^3$ being, respectively, a bounded subset of the Euclidean configuration space and the velocity space. Hence, f_{*s} varies slowly in time on the slow-time-scale $(\Delta t)^{eq}$, i.e. $\frac{d}{dt} \ln f_{*s}^{eq} \sim \frac{1}{(\Delta t)^{eq}}$. In view of the previous orderings holding for AD plasmas, this implies also $\frac{(\Delta t)^{eq}}{\tau_{col,s}} \ll 1$, where $\tau_{col,s}$ denotes the Spitzer collision time for the species s. Therefore, this requirement is consistent with the assumption of a collisionless plasma. A possible realization of f_{*s}^{eq} is provided by a non-isotropic generalized bi-Maxwellian KDF. As shown in Ref.[13], f_{*s}^{eq} determined in this way describes Vlasov-Maxwell equilibria characterized by quasi-neutral plasmas which exhibit species-dependent azimuthal and poloidal flows as well as temperature and pressure anisotropies. The existence of these equilibria is warranted by the validity of suitable kinetic constraints (see the discussion in Ref.[13, 15]). As a consequence, the same equilibria are characterized by the presence of fluid fields (number density, flow velocity, pressure tensor, etc.) which are generally non-uniform on the $\psi-$surfaces.

Let us now pose the problem of linear stability for Vlasov-Maxwell equilibria of this type. This can generally be set for perturbations of both the EM field and the equilibrium KDF, which exhibit appropriate time and space scales $\{(\Delta t)^{osc}, (\Delta L)^{osc}\}$. Here both are prescribed to have *fast time* and *fast space* dependencies with respect to those of the equilibrium quantities, in the sense that

$$\frac{(\Delta t)^{osc}}{(\Delta t)^{eq}} \sim \frac{(\Delta L)^{osc}}{(\Delta L)^{eq}} \sim O(\lambda), \tag{3}$$

with λ being a suitable infinitesimal parameter. In the case of strongly-magnetized AD plasmas, to permit a direct comparison with the literature, we also assume that these perturbations are *non-gyrokinetic.* In other words, they are characterized by typical wave-frequencies and wave-lengths which are much larger than the Larmor gyration frequency Ω_{cs} and radius r_{Ls}. This implies that the following inequalities must hold:

$$\frac{\tau_{Ls}}{(\Delta t)^{osc}} \sim \frac{r_{Ls}}{(\Delta L)^{osc}} \ll 1, \tag{4}$$

with $\tau_{Ls} = 1/\Omega_{cs}$, while λ must satisfy $\lambda \gg \sigma_s, \varepsilon_s, \varepsilon, \varepsilon_{M,s}$. These will be referred to as *low-frequency* and *long-wavelength perturbations* with respect to the corresponding Larmor scales. Notice that Eqs.(3) and (4) are independent and complementary, establishing the upper and lower limits for the range of magnitudes of both $(\Delta t)^{osc}$ and $(\Delta L)^{osc}$. We now determine the generic form of the perturbations as implied by the above assumptions. For this purpose, we shall require in the following that the EM field is subject to *axisymmetric EM perturbations* of the form $\delta\mathbf{B} = \nabla \times \delta\mathbf{A}$, $\delta\mathbf{E} = -\nabla\delta\phi - \frac{1}{c}\frac{\partial \delta\mathbf{A}}{\partial t}$, with $\left\{\delta\phi\left(\frac{\overline{\psi}}{\lambda}, \frac{\vartheta}{\lambda}, \frac{t}{\lambda}\right), \delta\mathbf{A}\left(\frac{\overline{\psi}}{\lambda}, \frac{\vartheta}{\lambda}, \frac{t}{\lambda}\right)\right\}$ both assumed to be *analytic* (with respect to $\overline{\psi}$ and ϑ) and *infinitesimal*, i.e., such that $\frac{\delta\mathbf{E}}{|\mathbf{E}^{(eq)}|}, \frac{\delta\mathbf{B}}{|\mathbf{B}^{(eq)}|} \sim O(\varepsilon)$. This implies that the corresponding perturbations for the EM potentials must scale as $\frac{\delta\phi}{|\Phi^{(eq)}|}, \frac{\delta\mathbf{A}}{|\mathbf{A}^{(eq)}|} \sim O(\varepsilon)O(\lambda)$, with $\mathbf{A}^{(eq)}$ denoting the equilibrium vector potential. As a consequence

$$\frac{d}{dt}E_s = q_s\left[\frac{\partial\delta\phi}{\partial t} - \frac{1}{c}\mathbf{v}\cdot\frac{\partial\delta\mathbf{A}}{\partial t}\right]. \tag{5}$$

Similarly, the perturbation of the equilibrium KDF is taken of the general form

$$\delta f_s \equiv \delta f_s\left(X_{*s}, (\psi_{*s}, \Phi_{*s}), \frac{\overline{\psi}}{\lambda}, \frac{\vartheta}{\lambda}, \frac{t}{\lambda}\right), \tag{6}$$

with $\frac{\delta f_s}{f_{*s}^{eq}} \sim O(\varepsilon)O(\lambda)$. It follows that the corresponding KDF (the solution of the Vlasov kinetic equation) must now be of the general form

$$f_s = f_s\left(X_{*s}, (\psi_{*s}, \Phi_{*s}), \frac{\overline{\psi}}{\lambda}, \frac{\vartheta}{\lambda}, \frac{t}{\lambda}\right), \tag{7}$$

while, from the Maxwell equations, the perturbations $\{\delta\phi, \delta\mathbf{A}\}$ are necessarily linear functionals of δf_s. However, for analytic perturbations of the form (7), f_s must itself be regarded as an analytic function of $\overline{\psi}$ and ϑ. Therefore, invoking Eqs.(1) and (2), the same KDF can always be considered as an asymptotic approximation obtained by Taylor expansion of a suitable *generalized KDF* of the form $f_s^{(gen)} \equiv f_s^{(gen)}\left(X_{*s}, (\psi_{*s}, \Phi_{*s}, Y_{*s}), \frac{t}{\lambda}\right)$, with $Y_{*s} \equiv \left[\frac{\varepsilon_s\psi_{*s}}{\lambda}, \frac{\sigma_s\Phi_{*s}}{\lambda}\right]$. In particular, denoting $\delta f_s^{(gen)} \equiv f_s^{(gen)} - f_{*s}^{eq}$, it follows that also $\delta f_s^{(gen)}$ is such that $\delta f_s^{(gen)} \equiv$

$\delta f_s^{(gen)}\left(X_{*s},(\psi_{*s},\Phi_{*s},Y_{*s}),\frac{t}{\lambda}\right)$. Then, by Taylor expansion with respect to the variables Y_{*s}, the perturbation $\delta f_s^{(gen)}$ can be shown to be related to δf_s (defined by Eq.(6)) by

$$\delta f_s^{(gen)} \cong \delta \widehat{f}_s\left(X_{*s},(\psi_{*s},\Phi_{*s}),\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right)\exp^{i\omega t}, \tag{8}$$

where corrections of $\frac{O(\varepsilon_s)}{O(\lambda)}$ and $\frac{O(\sigma_s)}{O(\lambda)}$ have been neglected. Similarly, invoking again Eqs.(1) and (2), for the analytic perturbations $\{\delta\phi,\delta\mathbf{A}\}$ we can introduce the corresponding *generalized perturbations* $\left\{\delta\phi^{(gen)},\delta\mathbf{A}^{(gen)}\right\}$. Neglecting in the similar way corrections of $\frac{O(\varepsilon_s)}{O(\lambda)}$ and $\frac{O(\sigma_s)}{O(\lambda)}$, these are given as follows:

$$\delta\phi^{(gen)}\left(Y_{*s},\frac{t}{\lambda}\right) \;\cong\; \delta\widehat{\phi}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right)\exp^{i\omega t}, \tag{9}$$

$$\delta\mathbf{A}^{(gen)}\left(Y_{*s},\frac{t}{\lambda}\right) \;\cong\; \delta\widehat{\mathbf{A}}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right)\exp^{i\omega t}. \tag{10}$$

Analogous expressions for the corresponding generalized perturbations can be readily obtained. In particular, using Eq.(8), we get the following representation for $\delta f_s^{(gen)}$:

$$\delta f_s^{(gen)} = \delta\widehat{f}_s^{(gen)}\left(X_{*s},(\psi_{*s},\Phi_{*s},Y_{*s})\right)\exp^{i\omega t}, \tag{11}$$

where, expanding the Fourier coefficient and neglecting again corrections of $\frac{O(\varepsilon_s)}{O(\lambda)}$ and $\frac{O(\sigma_s)}{O(\lambda)}$, $\delta\widehat{f}_s^{(gen)} \cong \delta\widehat{f}_s\left(X_{*s},(\psi_{*s},\Phi_{*s}),\frac{\varepsilon\psi}{\lambda},\frac{\vartheta}{\lambda}\right)$. Therefore, in view of Eq.(5), for infinitesimal axisymmetric analytical EM perturbations $\{\delta\phi,\delta\mathbf{A}\}$, to leading order in λ the Vlasov equation implies the dispersion equation

$$\begin{aligned} -i\omega\delta\widehat{f}_s\left(X_{*s},(\psi_{*s},\Phi_{*s}),\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right) = \\ = i\omega q_s\left[\delta\widehat{\phi}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right)-\frac{1}{c}\mathbf{v}\cdot\delta\widehat{\mathbf{A}}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right)\right]\frac{\partial f_{*s}^{eq}}{\partial E_s}. \end{aligned} \tag{12}$$

Apart from the trivial solution $\omega=0$ (i.e., a stationary perturbation of the equilibrium), this requires that, for $\omega\neq 0$, one must have

$$\delta\widehat{f}_s = -q_s\left[\delta\widehat{\phi}-\frac{1}{c}\mathbf{v}\cdot\delta\widehat{\mathbf{A}}\right]\frac{\partial f_s^{(eq)}}{\partial E_s}, \tag{13}$$

where, by construction, $\delta\widehat{f}_s$, $\delta\widehat{\phi}$ and $\delta\widehat{\mathbf{A}}$ are manifestly independent of ω. Hence, Eq.(13) necessarily holds also when $|\omega|$ is arbitrarily small. In this limit $\left\{\delta\widehat{\phi},\delta\widehat{\mathbf{A}},\delta\widehat{f}_s\right\}$ tend necessarily to infinitesimal stationary perturbations of the equilibrium solutions. On the other hand, Eqs.(9), (10) and (11) show that $\left\{\delta\widehat{\phi},\delta\widehat{\mathbf{A}},\delta\widehat{f}_s\right\}$ are always asymptotically close to the generalized quantities $\left\{\delta\widehat{\phi}^{(gen)},\delta\widehat{\mathbf{A}}^{(gen)},\delta\widehat{f}_s^{(gen)}\right\}$, which are by definition equilibrium perturbations [i.e., functions of $\left(\frac{\varepsilon_s\psi_{*s}}{\lambda},\frac{\sigma_s\Phi_{*s}}{\lambda}\right)$]. Since the latter again represent an equilibrium and are independent of ω, it follows that the only admissible solution of the dispersion equation (13) is clearly independent of ω as well and coincides with the null solution, i.e.

$$\delta\widehat{\phi}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right) \;\equiv\; 0, \tag{14}$$

$$\delta\widehat{\mathbf{A}}\left(\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right) \;\equiv\; 0, \tag{15}$$

$$\delta\widehat{f}_s\left(X_{*s},(\psi_{*s},\Phi_{*s}),\frac{\overline{\psi}}{\lambda},\frac{\vartheta}{\lambda}\right) \;\equiv\; 0. \tag{16}$$

In summary: *no analytic, low-frequency and long-wavelength axisymmetric unstable perturbations can exist in non-relativistic strongly-magnetized and gravitationally-bound axisymmetric collisionless AD plasmas.* We stress that this

result follows from two basic assumptions. The first one is the requirement that the equilibrium magnetic field admits locally nested $\psi-$surfaces. The second one is due to the assumed property of AD plasmas to be gravitationally-bound. This implies (as pointed out above) that the effective ES potential Φ_s^{eff} is necessarily a function of both ψ and ϑ, and therefore the perturbation of the KDF is actually close to a function of the exact and adiabatic invariants X_{*s}. A notable aspect of the conclusion is that it applies to collisionless Vlasov-Maxwell equilibria having, in principle, arbitrary topology of the magnetic field lines which can belong to either closed or open magnetic $\psi-$surfaces. Also, as pointed out in Refs.[12, 13], for strongly-magnetized plasmas these equilibria can give rise to kinetic dynamo effects simultaneously with having accretion flows. These results are important for understanding the phenomenology of collisionless AD plasmas of this type. In particular, they completely rule out the possibility that axisymmetric perturbations, which are long-wavelength and low-frequency in the sense of the inequalities (4), could give rise to kinetic instabilities in such systems. This conclusion applies for collisionless AD plasmas (having in particular particle densities within the range mentioned earlier) which are strongly-magnetized and simultaneously gravitationally-bound . Since fluid descriptions of these plasmas can only be arrived at on the basis of the present Vlasov-Maxwell statistical description, also MHD instabilities, such as the axisymmetric MRI [2, 20], the axisymmetric TMI (see for example [9–11]), and axisymmetric instabilities driven by temperature anisotropy (e.g., the firehose instability [21]) remain definitely forbidden for collisionless plasmas under these conditions.

Acknowledgments - This work has been partly developed in the framework of MIUR (Italian Ministry of University and Research) PRIN Research Programs and the Consortium for Magnetofluid Dynamics, Trieste, Italy.

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